

C & EE 141

Bending Members
Laterally Unsupported and
Noncompact

Limit States for Flexure

- Plastic Flexural Capacity
- Shear Capacity
- Deflection

- *Only limited to these three limit states when compression flange is laterally braced and beam is compact.*

**What if the compression flange isn't braced?
What if there is a local buckling issue?**

Limit States for Flexure

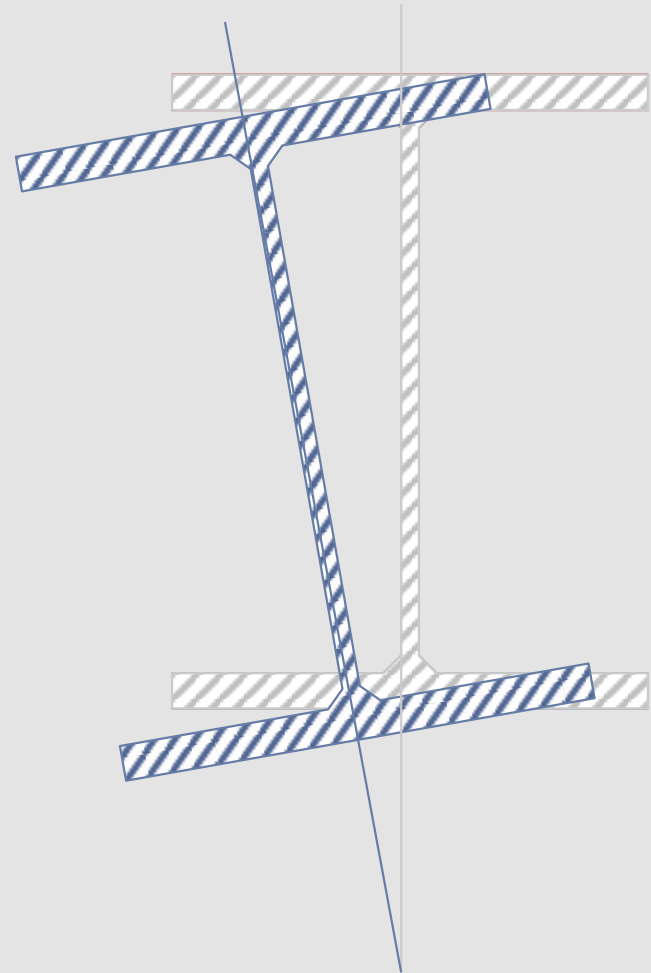
- Plastic Flexural Capacity
- **Global Buckling**
 - Inelastic lateral torsional buckling
 - Elastic lateral torsional buckling
- **Local Buckling**
 - Width-thickness ratios for web and
- Shear Capacity
- Deflection

Limit States for Flexure

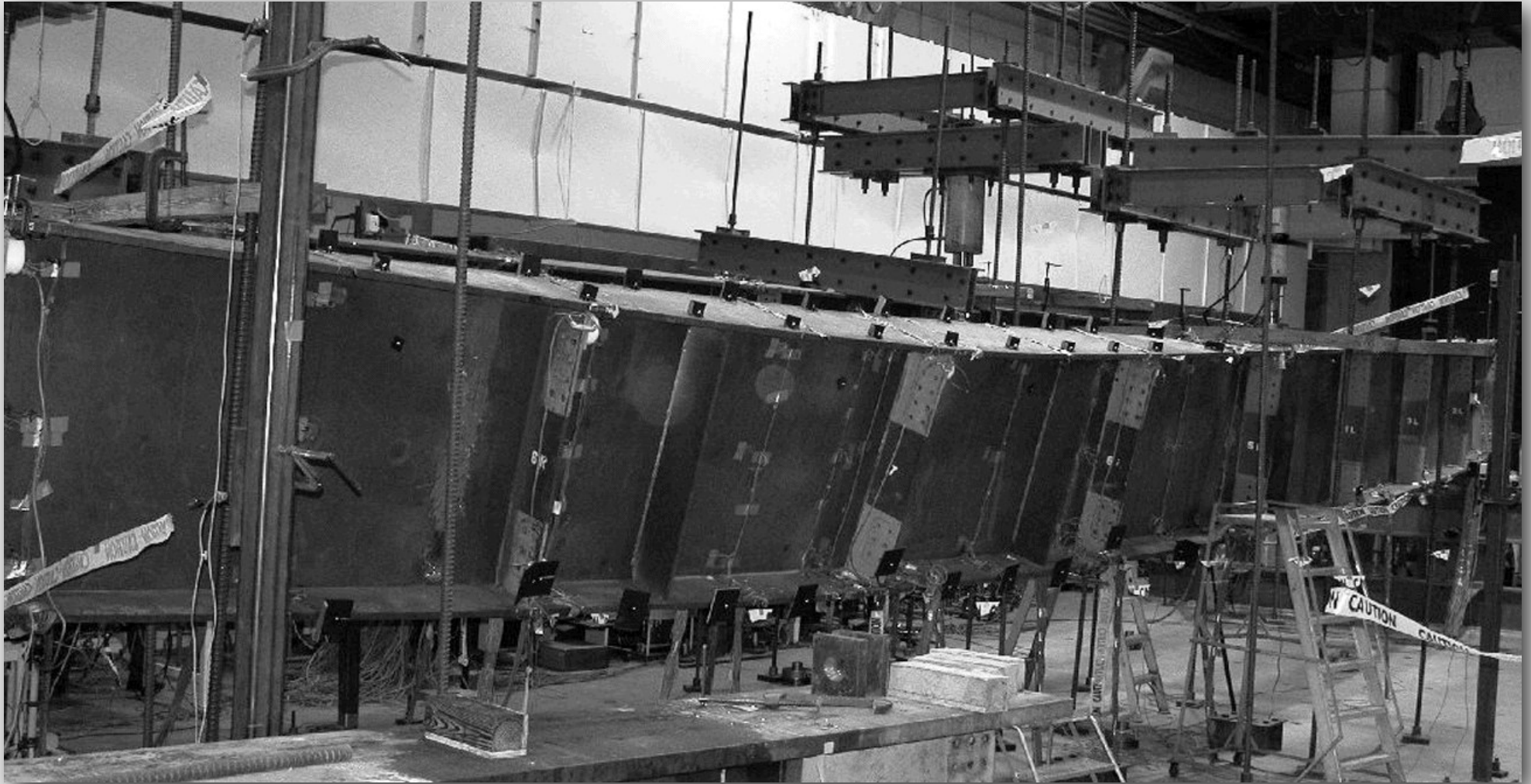
- Plastic Flexural Capacity
- Global Buckling
 - Inelastic lateral torsional buckling
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What is Lateral Torsional Buckling?

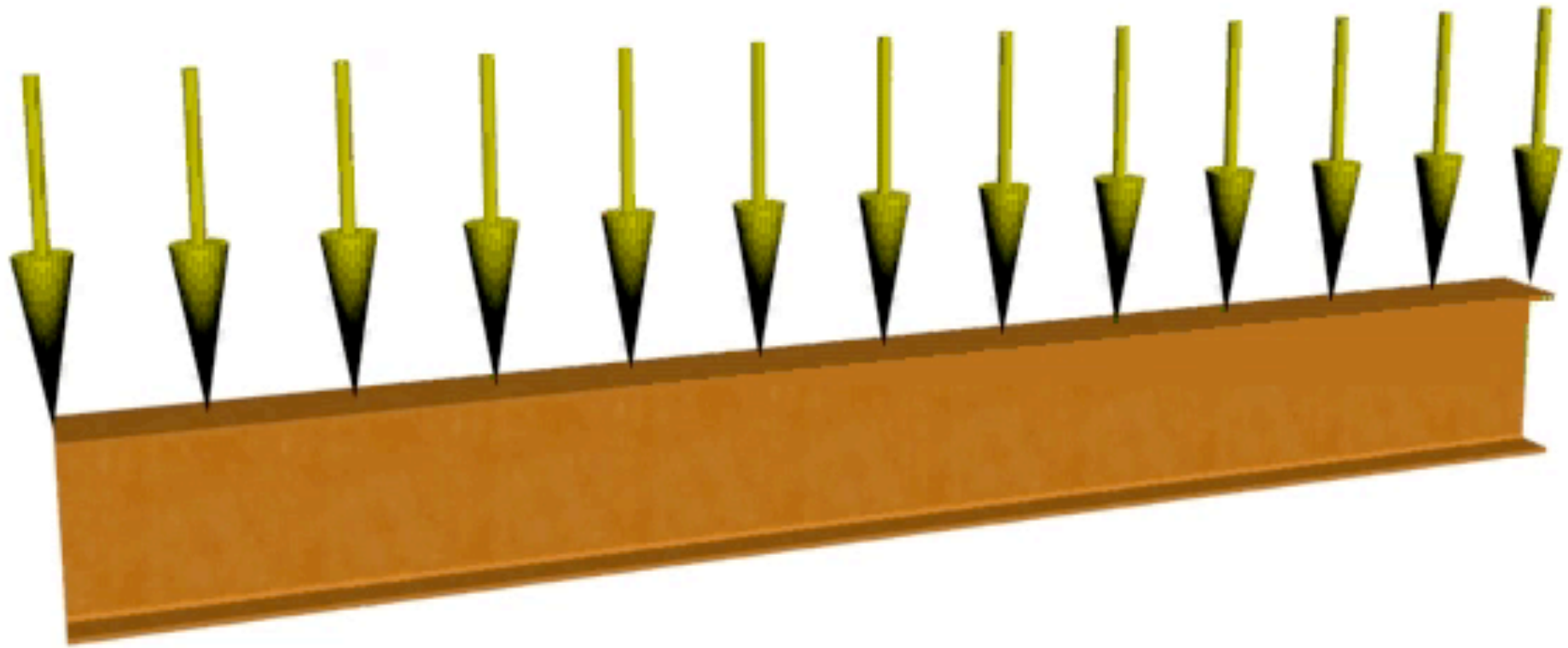
- The compression zone of the section buckles if not restrained
- For a simply supported WF that is the top flange and, to a varying extent, the web
- For a cantilevered WF that is the bottom flange and, to a varying extent, the web



What is Lateral Torsional Buckling?



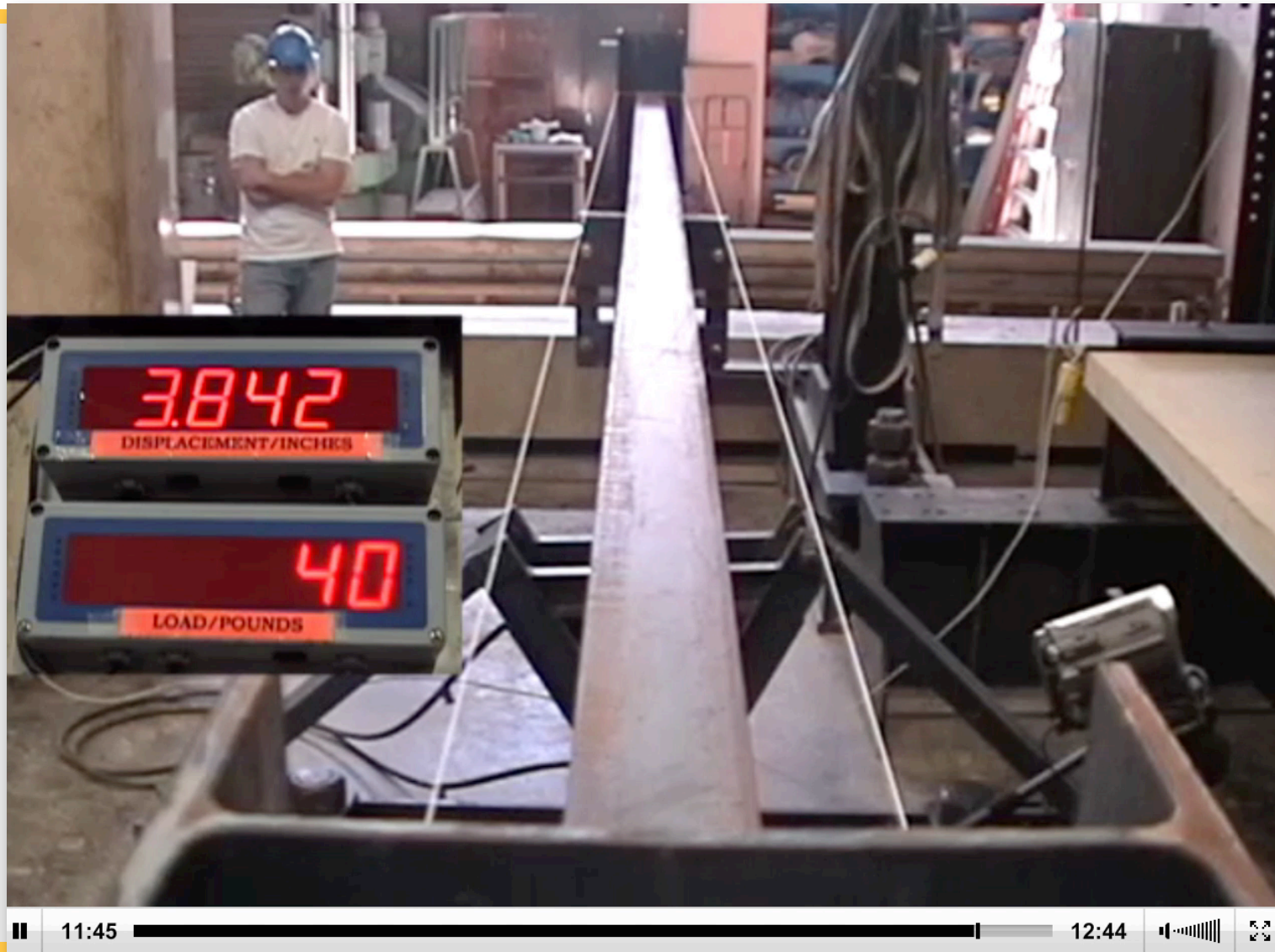
Beams experience Lateral Torsional Buckling (LTB), due to buckling of the compression flange of a beam



Onset of Lateral Torsional Buckling

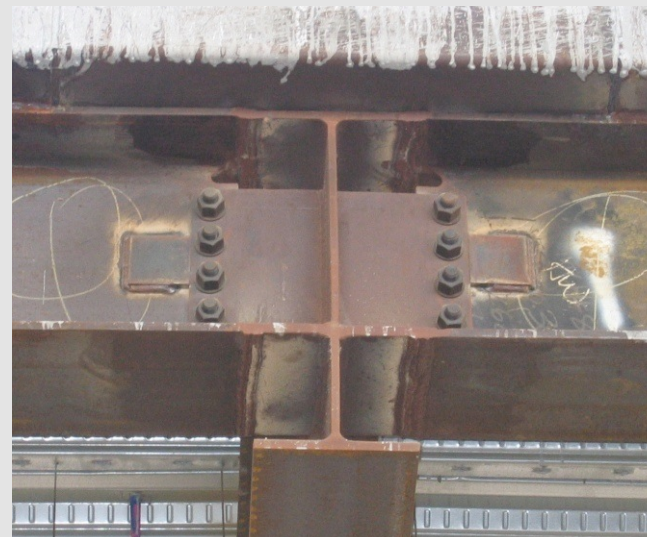


Elastic LTB



How is Lateral Torsional Buckling Prevented?

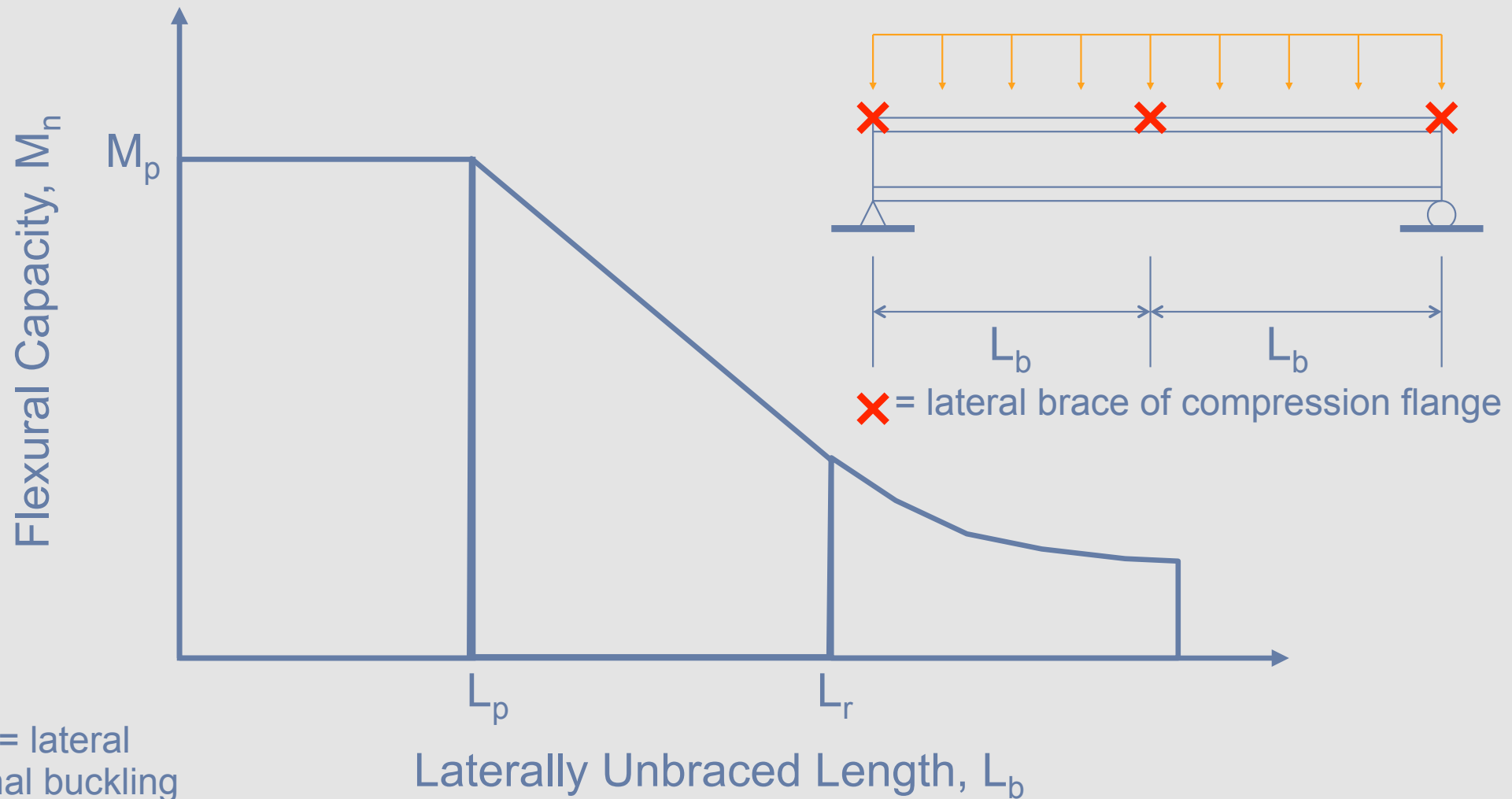
- Headed steel studs welded to the beam flange harden in the concrete fill which provides continuous stability
- Connections from perpendicular framing beams provide discrete points of stability



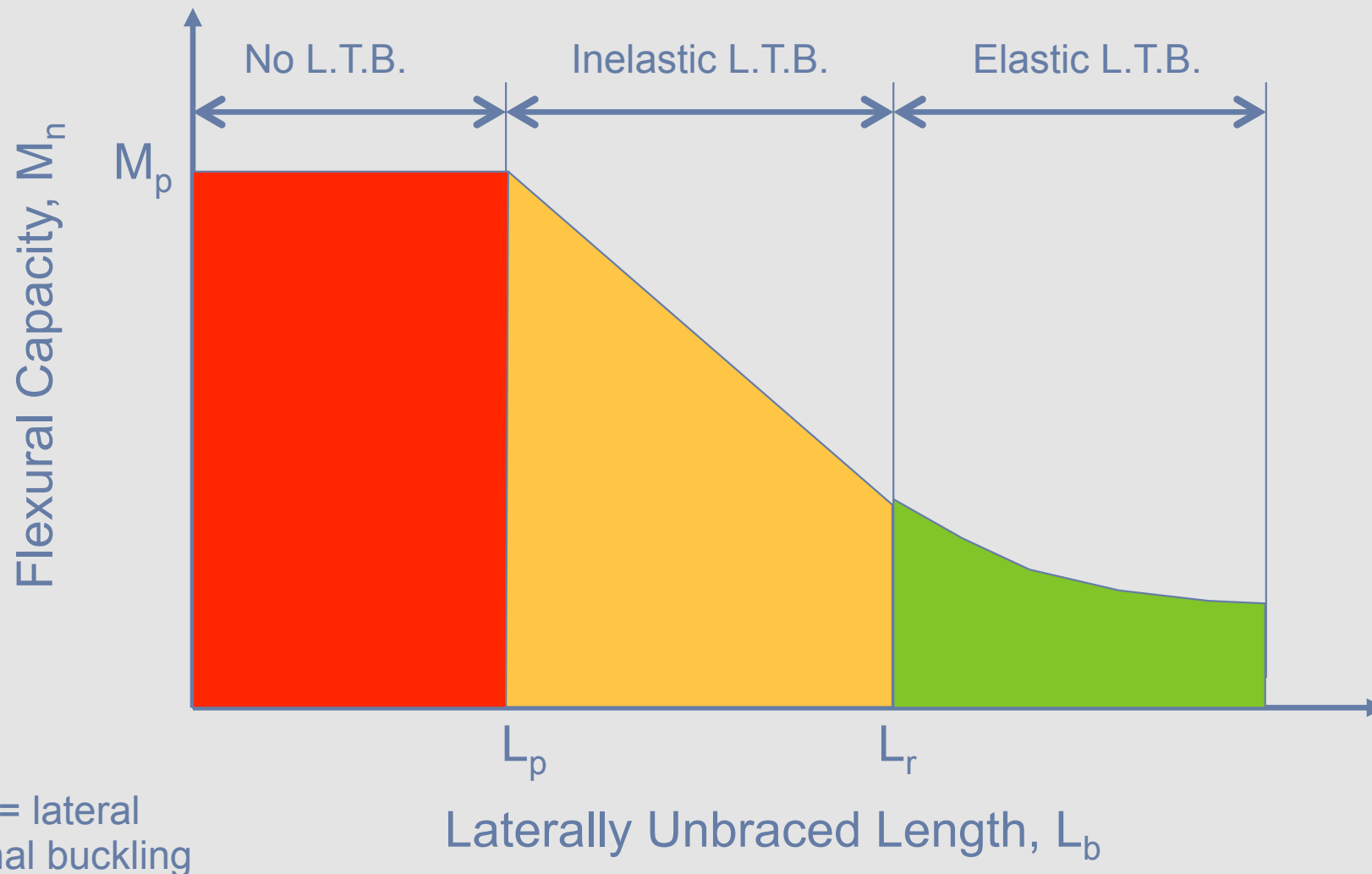
Example of an Unbraced Beam



Transitions in Flexural Capacity due to Global Buckling



Transitions in Flexural Capacity due to Global Buckling



F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric I-shaped members and channels bent about their major axis, having compact webs and compact flanges as defined in Section B4.1 for flexure.

User Note: All current ASTM A6 W, S, M, C and MC shapes except W21×48, W14×99, W14×90, W12×65, W10×12, W8×31, W8×10, W6×15, W6×9, W6×8.5 and M4×6 have compact flanges for $F_y = 50$ ksi (345 MPa); all current ASTM A6 W, S, M, HP, C and MC shapes have compact webs at $F_y \leq 65$ ksi (450 MPa).

The *nominal flexural strength*, M_n , shall be the lower value obtained according to the limit states of yielding (plastic moment) and lateral-torsional buckling.

1. Yielding

$$M_n = M_p = F_y Z_x \quad (\text{F2-1})$$

where

F_y = specified minimum yield stress of the type of steel being used, ksi (MPa)

Z_x = plastic section modulus about the x -axis, in.³ (mm³)

2. Lateral-Torsional Buckling

- (a) When $L_b \leq L_p$, the *limit state of lateral-torsional buckling* does not apply.
- (b) When $L_p < L_b \leq L_r$

$$M_n = C_b \left[M_p - (M_p - 0.7F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \quad (\text{F2-2})$$

- (c) When $L_b > L_r$

$$M_n = F_{cr} S_x \leq M_p \quad (\text{F2-3})$$

where

L_b = length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section, in. (mm)

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}} \right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}} \right)^2} \quad (\text{F2-4})$$

and where

E = modulus of elasticity of steel = 29,000 ksi (200 000 MPa)

J = torsional constant, in.⁴ (mm⁴)

S_x = elastic section modulus taken about the x -axis, in.³ (mm³)

h_o = distance between the flange centroids, in. (mm)

The limiting lengths L_p and L_r are determined as follows:

$$L_p = 1.76r_y \sqrt{\frac{E}{F_y}} \quad (\text{F2-5})$$

$$L_r = 1.95r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76 \left(\frac{0.7F_y}{E}\right)^2}} \quad (\text{F2-6})$$

where

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \quad (\text{F2-7})$$

and the coefficient c is determined as follows:

(a) For doubly symmetric I-shapes: $c = 1$ (F2-8a)

(b) For channels: $c = \frac{h_o}{2} \sqrt{\frac{I_y}{C_w}}$ (F2-8b)

User Note: The square root term in Equation F2-4 may be conservatively taken equal to 1.0.

User Note: Equations F2-3 and F2-4 provide identical solutions to the following expression for lateral-torsional buckling of doubly symmetric sections that has been presented in past editions of the AISC LRFD Specification:

$$M_{cr} = C_b \frac{\pi}{L_b} \sqrt{EI_y GJ + \left(\frac{\pi E}{L_b} \right)^2 I_y C_w}$$

The advantage of Equations F2-3 and F2-4 is that the form is very similar to the expression for lateral-torsional buckling of singly symmetric sections given in Equations F4-4 and F4-5.

User Note: For doubly symmetric I-shapes with rectangular flanges, $C_w = \frac{I_y h_o^2}{4}$ and thus Equation F2-7 becomes

$$r_{ts}^2 = \frac{I_y h_o}{2S_x}$$

r_{ts} may be approximated accurately and conservatively as the radius of gyration of the compression flange plus one-sixth of the web:

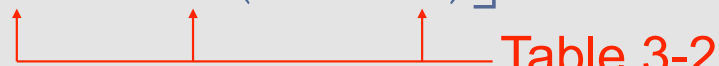
$$r_{ts} = \frac{b_f}{\sqrt{12 \left(1 + \frac{1}{6} \frac{h t_w}{b_f t_f} \right)}}$$

Global Buckling Limits

1. Inelastic LTB ($L_p < L_b \leq L_r$)

$$M_n = C_b \left[M_p - (M_p - 0.7 F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \quad (\text{Eq F2-2})$$

$$\phi_b M_n = C_b \left[\phi_b M_p - \phi_b B F (L_b - L_p) \right] \leq \phi_b M_p \quad (\text{pg 3-9})$$

 Table 3-2

2. Elastic LTB ($L_b > L_r$)

$$M_n = F_{cr} S_x \leq M_p \quad (\text{Eq F2-3})$$

where F_{cr} per Eq F2-4

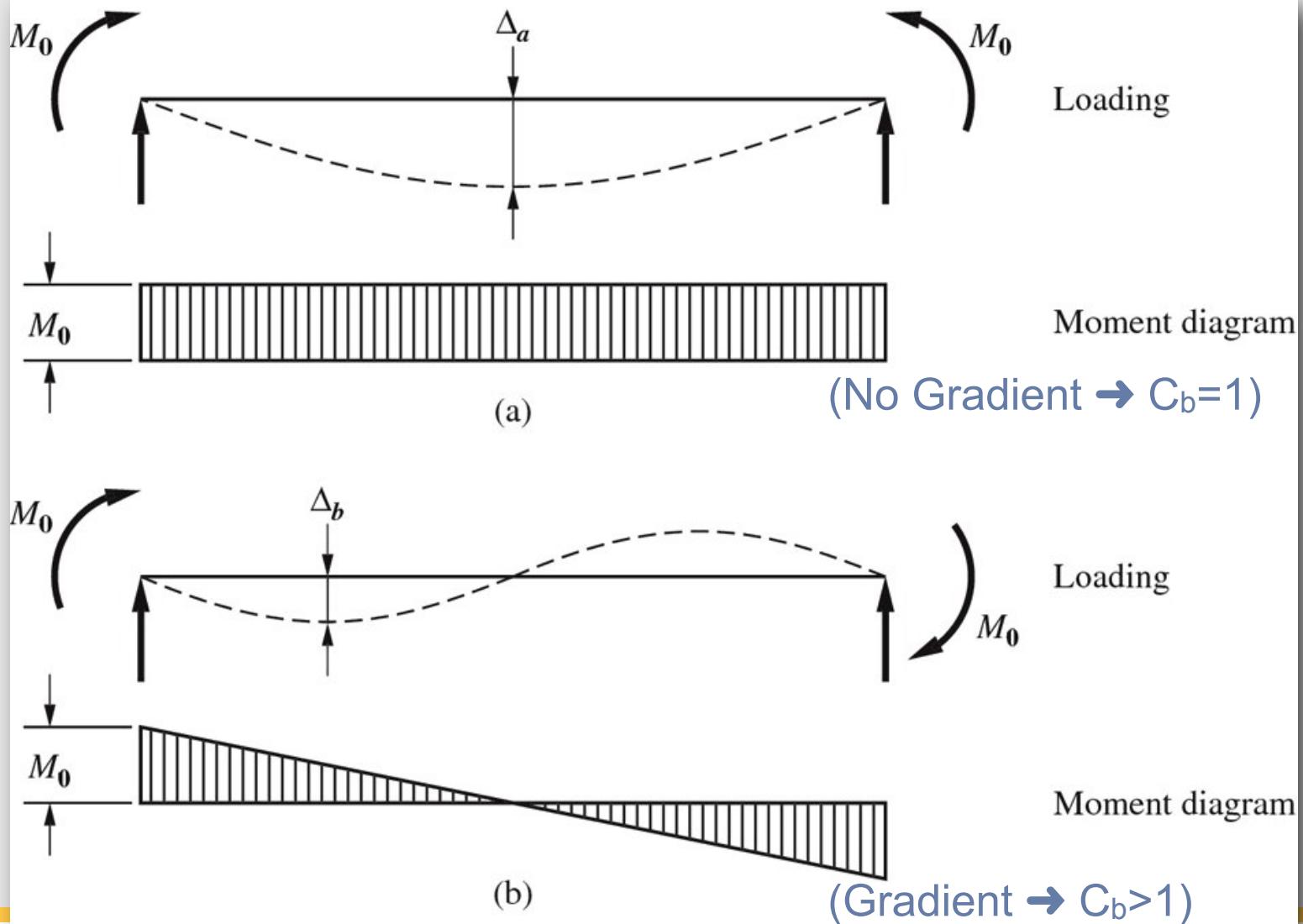
$$F_y = 50 \text{ ksi}$$
$$\mathbf{Z}_x$$

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in. ³	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in. ⁴	ASD	LRFD
W30×116	378	943	1420	575	864	24.8	37.4	7.74	22.6	4930	339	509
W21×147	373	931	1400	575	864	13.7	20.7	10.4	36.3	3630	318	477
W24×131	370	923	1390	575	864	16.3	24.6	10.5	31.9	4020	296	445
W18×158	356	888	1340	541	814	10.5	15.9	9.68	42.8	3060	319	479
W14×193	355	886	1330	541	814	5.30	7.93	14.3	79.4	2400	276	414
W12×210	348	868	1310	510	767	4.25	6.45	11.6	95.8	2140	347	520
W30×108	346	863	1300	522	785	23.5	35.5	7.59	22.1	4470	325	487
W27×114	343	856	1290	522	785	21.7	32.8	7.70	23.1	4080	311	467
W21×132	333	831	1250	515	774	13.2	19.9	10.3	34.2	3220	283	425
W24×117	327	816	1230	508	764	15.4	23.3	10.4	30.4	3540	267	401
W18×143	322	803	1210	493	740	10.3	15.7	9.61	39.6	2750	285	427
W14×176	320	798	1200	491	738	5.20	7.83	14.2	73.2	2140	252	378
ASD	LRFD	v Shape does not meet the h/t_w limit for shear in AISC Specification Section G2.1(a) with $F_y = 50$ ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.										
$\Omega_b = 1.67$ $\Omega_v = 1.50$	$\phi_b = 0.90$ $\phi_v = 1.00$											

The Moment Gradient (C_b)

- C_b is a coefficient used to account for the effect of moment gradients on LTB
- End restraints change the moment diagram of the beam so that the effective length of the “compression” element of a beam may change
- C_b is needed because all calculations and tables are based on a case with uniform moment, where $C_b = 1.0$

The Moment Gradient (C_b)



The Moment Gradient (C_b)

(3) For singly symmetric members in *single curvature* and all doubly symmetric members:

C_b , the *lateral-torsional buckling* modification factor for nonuniform moment diagrams when both ends of the segment are braced is determined as follows:

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C} \quad (F1-1)$$

where

M_{max} = absolute value of maximum moment in the unbraced segment, kip-in. (N-mm)

M_A = absolute value of moment at quarter point of the unbraced segment, kip-in. (N-mm)

M_B = absolute value of moment at centerline of the unbraced segment, kip-in. (N-mm)

M_C = absolute value of moment at three-quarter point of the unbraced segment, kip-in. (N-mm)

For cantilevers or overhangs where the free end is unbraced, $C_b = 1.0$.

The Moment Gradient (C_b)

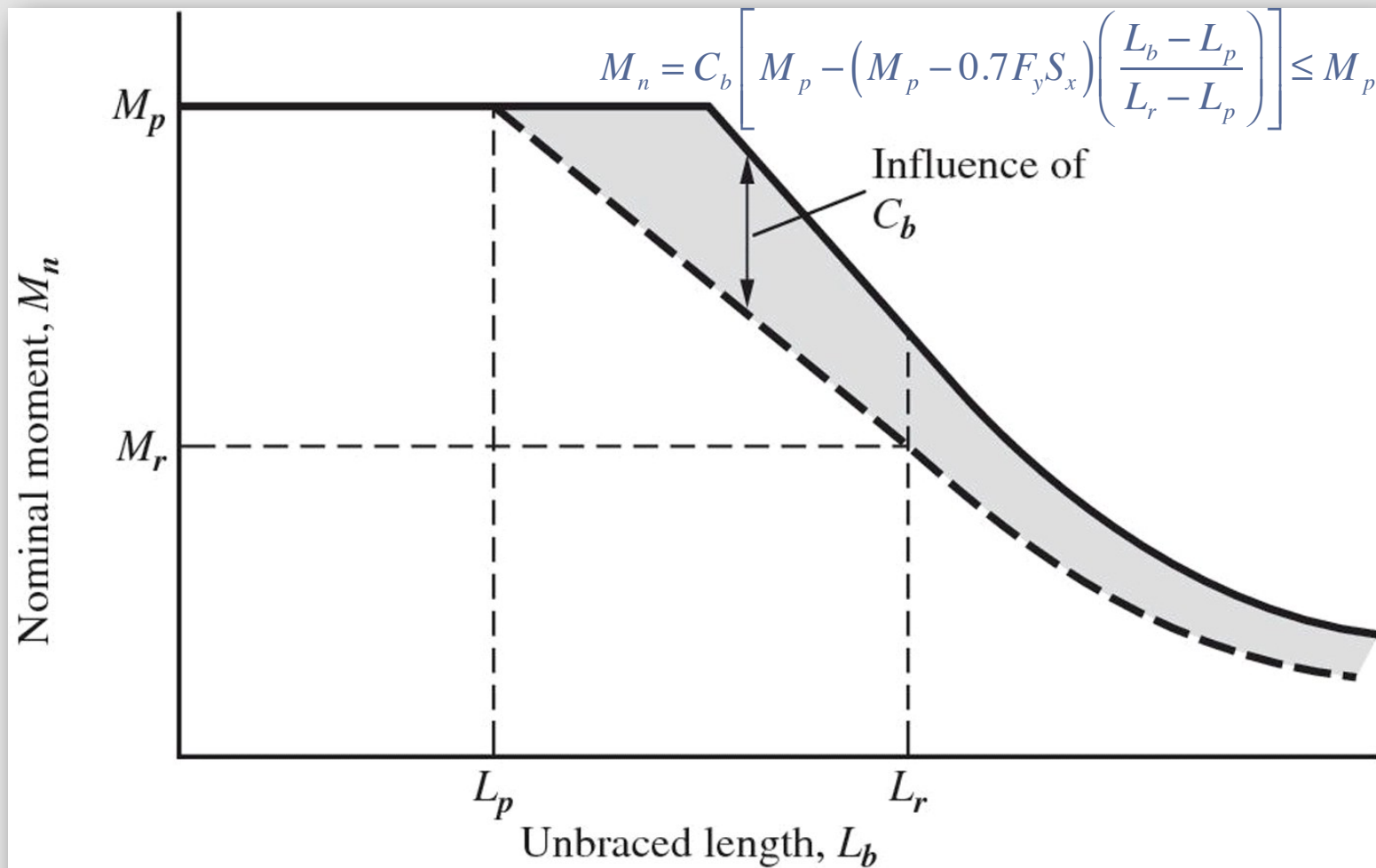
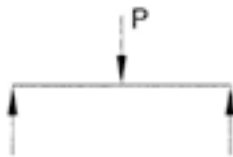
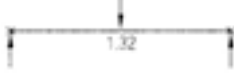
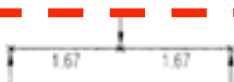



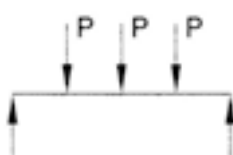


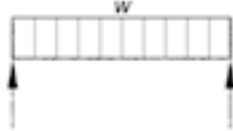
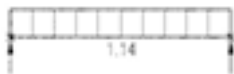
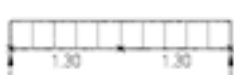

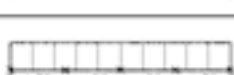

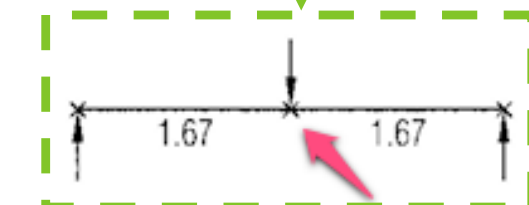
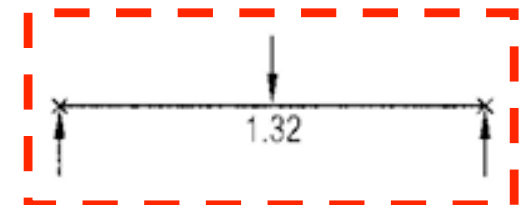


Figure 6.14
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Table 3-1
Values of C_b for Simply Supported Beams

Load	Lateral Bracing Along Span	C_b
	None Load at midpoint	
	At load point	
	None Loads at third points	
	At load points Loads symmetrically placed	
	None Loads at quarter points	
	At load points Loads at quarter points	
	None	
	At midpoint	
	At third points	
	At quarter points	
	At fifth points	

Note: Lateral bracing must always be provided at points of support per AISC Specification Chapter F.



The little "x" indicates a lateral bracing point

Limit States for Flexure

- Plastic Flexural Capacity
- Global Buckling
 - Inelastic lateral torsional buckling
 - Elastic lateral torsional buckling
- Local Buckling
 - Width-thickness ratios for web and flanges
- Shear Capacity
- Deflection

Local Instability

- Buckling of one of the compression elements of the x-section.
 - Flange Local Buckling
 - Web Local Buckling

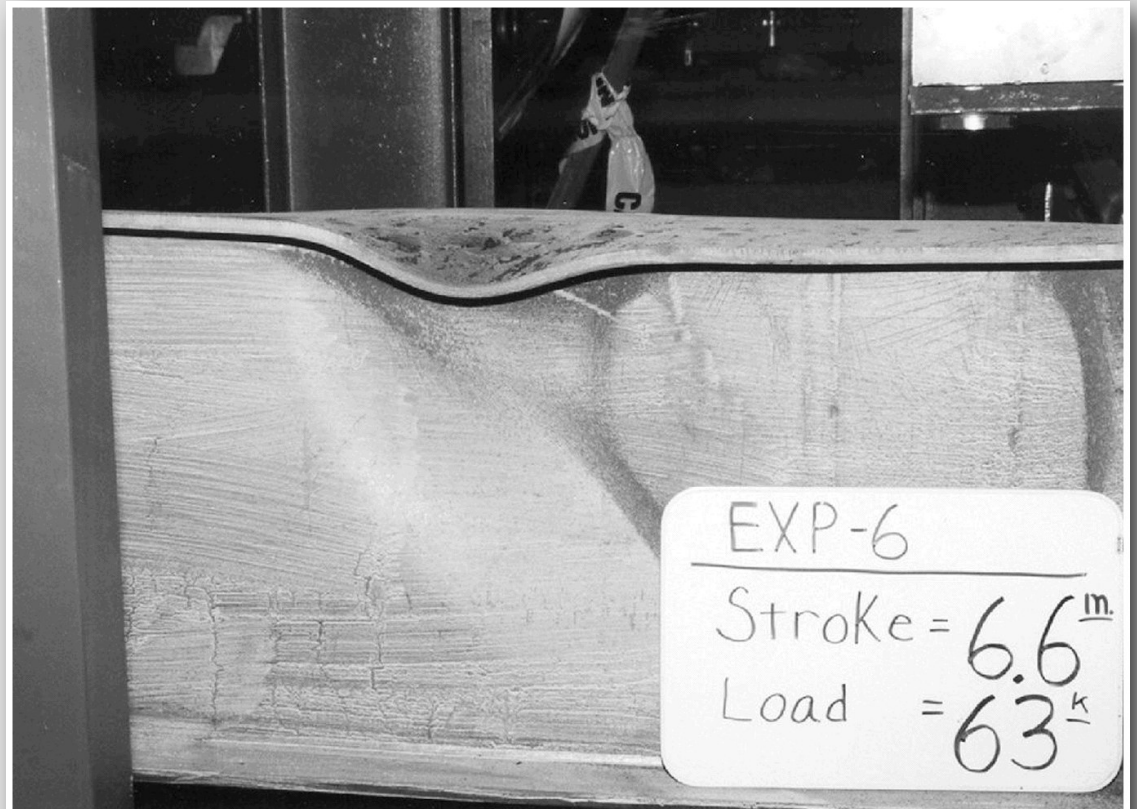
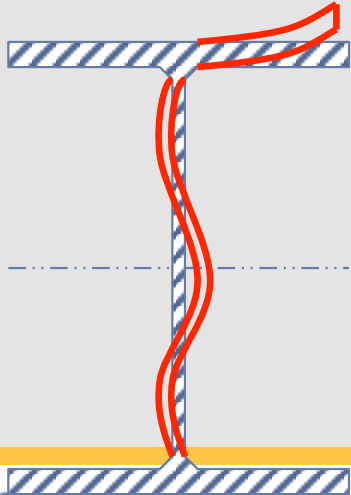


Figure 6.16
Photo courtesy of Donald W. White

Local Instability

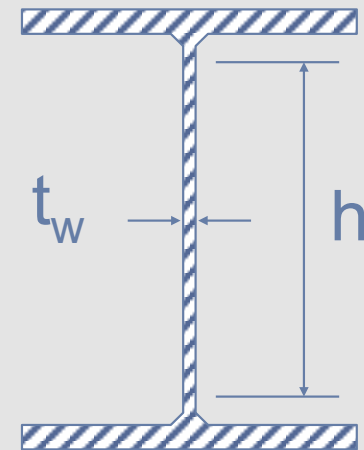
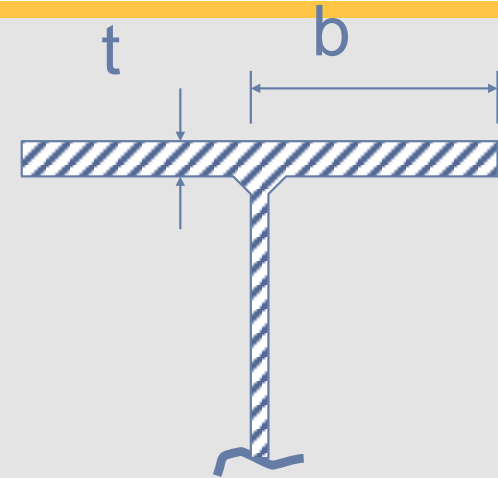
- Whether a member is subject to a local instability is a function of the width-thickness ratio of the elements that can buckle:

- Flange Local Buckling

$$\lambda = b/t$$

- Web Local Buckling

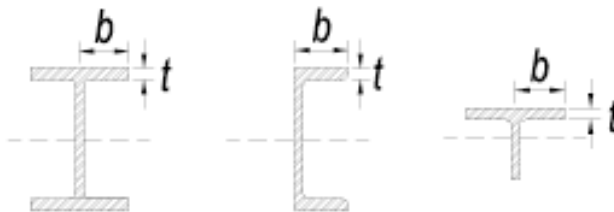
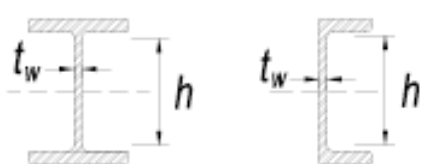
$$\lambda = h/t_w$$



Checking the Ratios

- Limiting width-thickness ratios are function of:
 - geometry of the shapes
 - yield stress of the steel
- Shapes can be classified as:
 - A **compact shape** is not subject to local buckling.
 - A **non-compact shape** is subject to inelastic local buckling after initial yielding.
 - A **slender shape** is subject to elastic L.B.
- Local buckling will limit the moment capacity that can be reached by the member, regardless of L_b

Classification of Sections for Local Buckling

Case	Description of Element	(λ) Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio		Examples
			λ_p (compact/noncompact)	λ_r (noncompact/slender)	
10	Flanges of rolled I-shaped sections, channels, and tees	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$	
15	Webs of doubly-symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$	

Spec Table B4.1b

Effect on Flexural Capacity due to Local Buckling

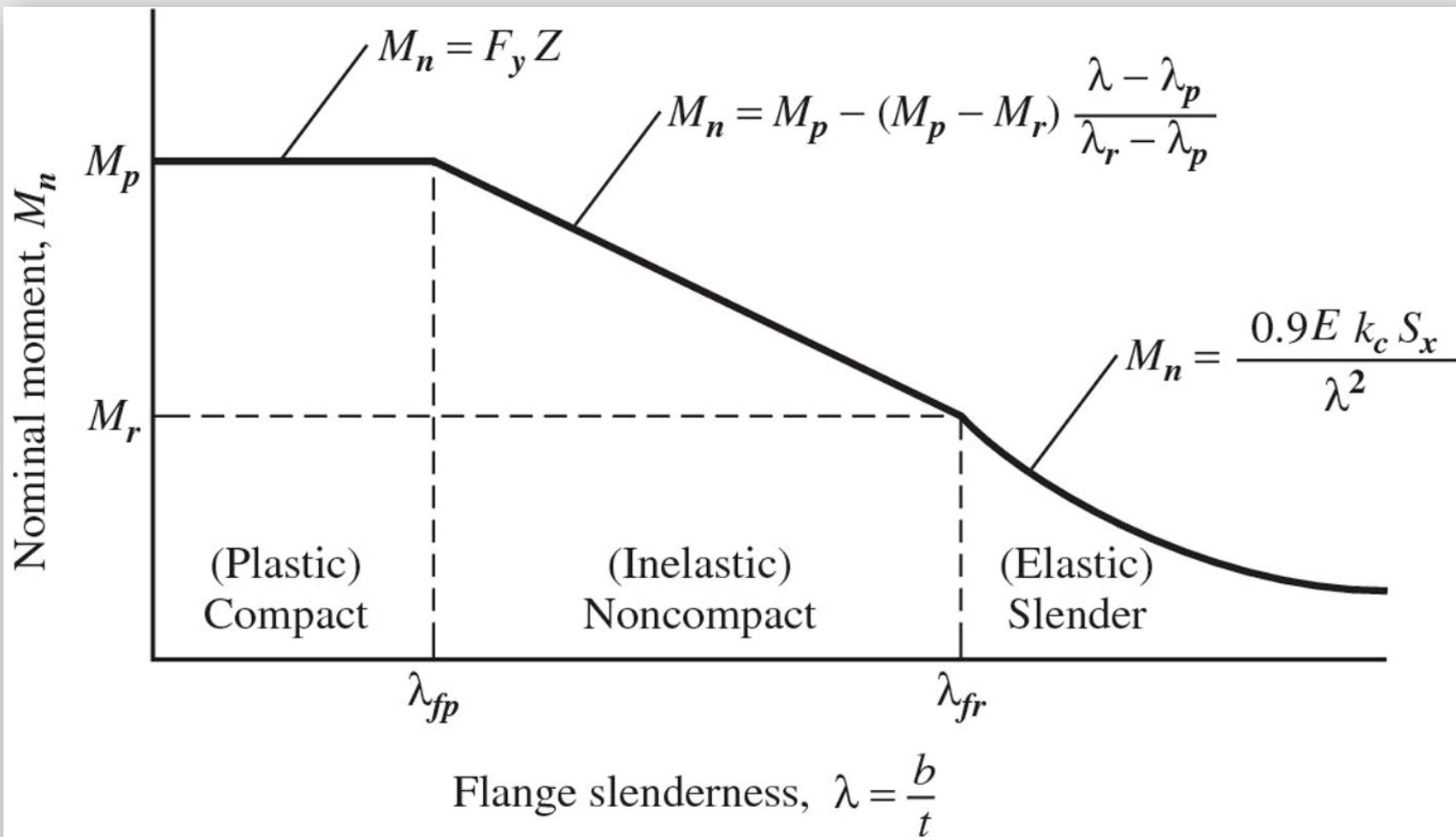


Figure 6.18

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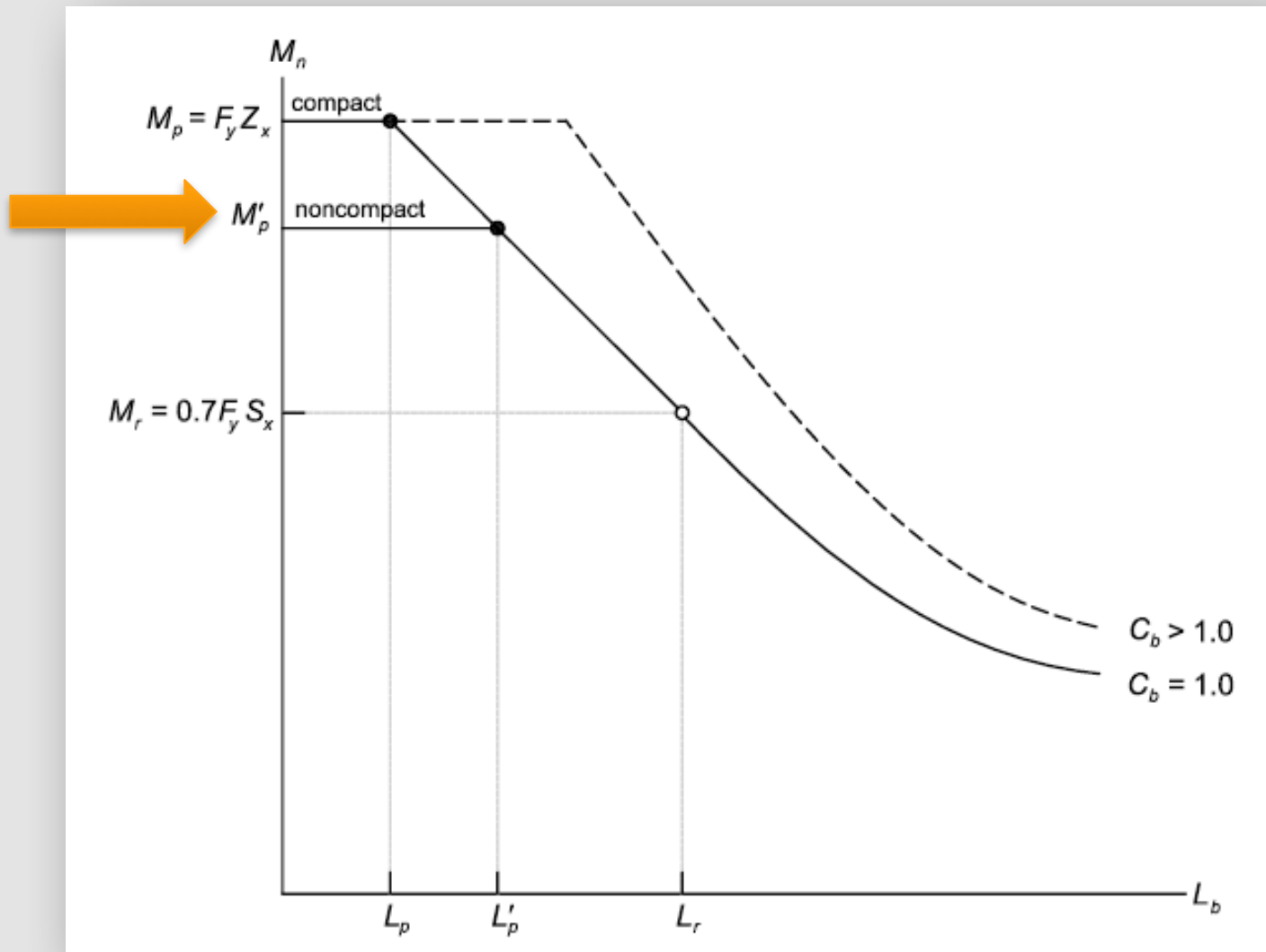
Maximum Flexural Capacity for Noncompact Sections

- Maximum moment capacity limited, regardless of LTB capacity:
 - For WF beam with noncompact flanges:

$$M_n = M_p - \left(M_p - 0.7 F_y S_x \right) \frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}$$

- WF beams with slender flanges, or noncompact/slender webs trigger additional checks. (Not covered in this class).

Effect of Noncompact Section on Moment Capacity



Design Checks for Flexure

1. Lateral Torsional Buckling
 - Checked for adequate lateral support of the compression flange.
2. Flange Local Buckling
 - Checked with $b_f/2t_f$ ratio.
3. Web Local Buckling
 - Checked with h/t_w ratio.
4. Formation of Plastic Moment
 - $M_u \leq \phi_b M_p$

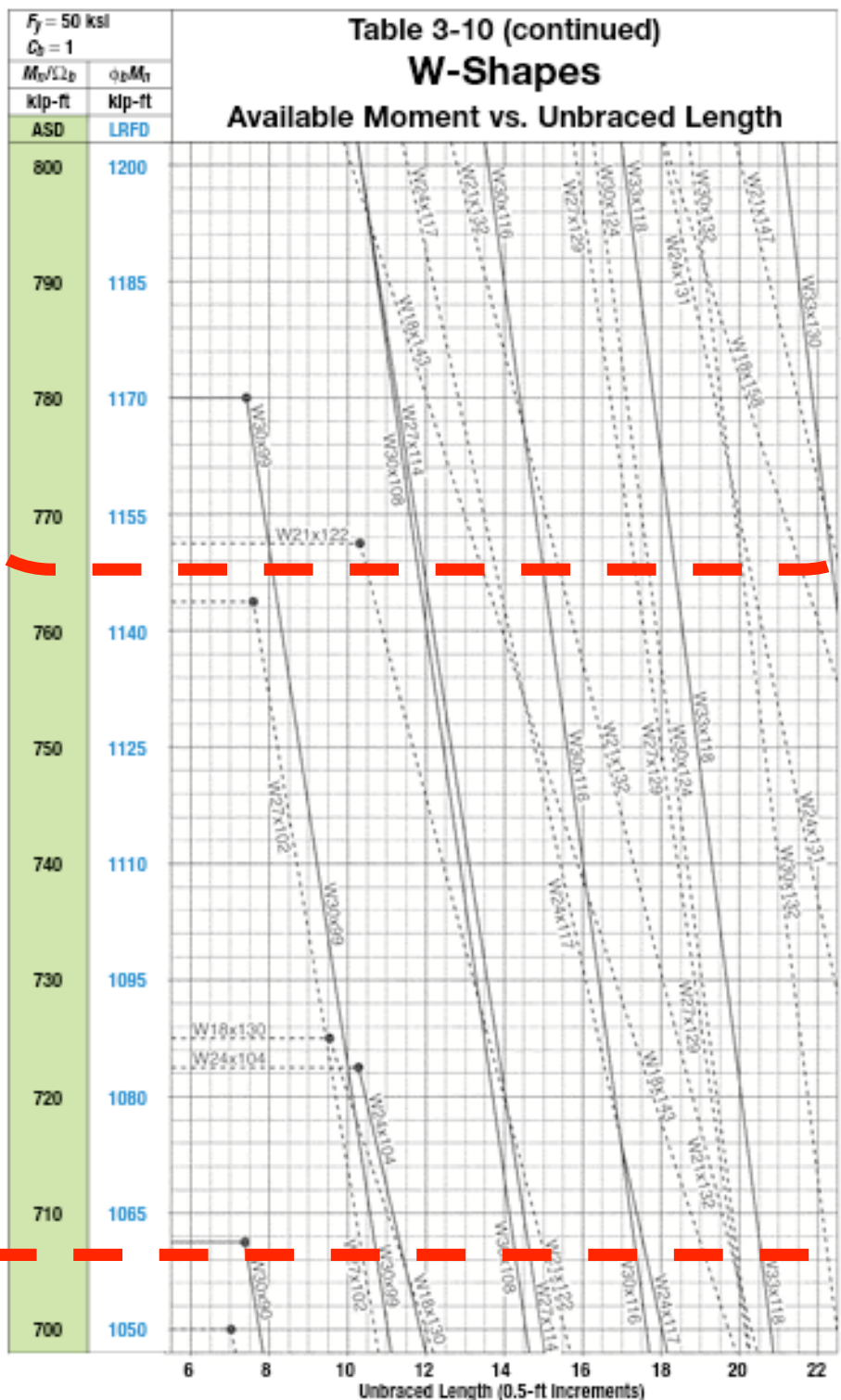
Do We Need All These Equations?

- Refer to AISC
 - Section Properties
 - Table 1-1: width-thickness ratios, S , I , Z
 - Beam Tables
 - Table 3-2: select most economical size based on Z_x
 - Table 3-3: select most economical size based on I_x
 - Table 3-10: select most economical size for particular unbraced length
 - Note: compactness of section is considered in tables

Beam Tables Based on Strong Axis Strength & Unbraced Length

- Actual unbraced length (L_b) of the beam is accounted for
- Very commonly used table

Available Moment, M_u/Ω_b (2 kip-ft increments) and $\phi_b M_u$ (3 kip-ft increments)





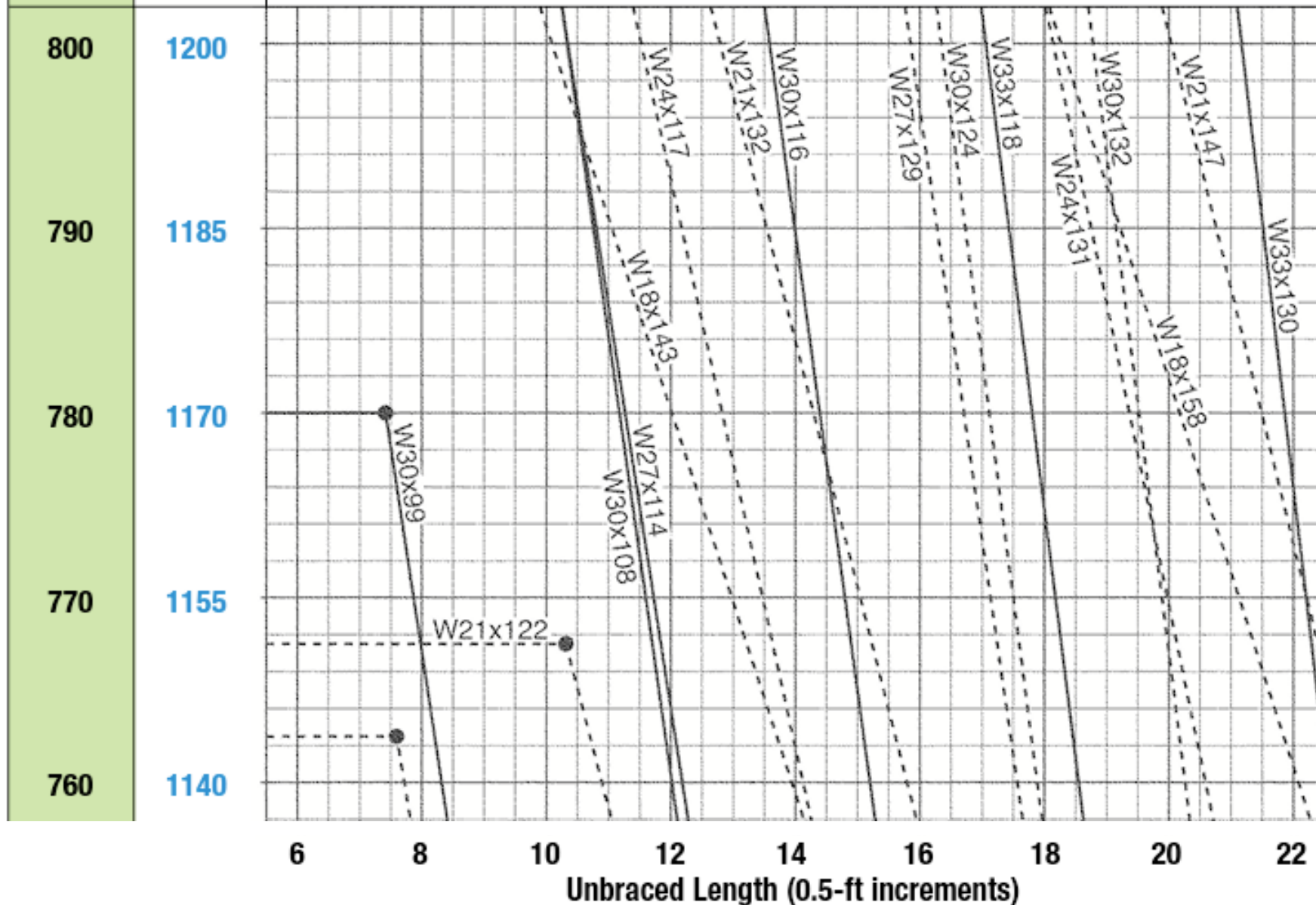
$F_y = 50 \text{ ksi}$	
$C_b = 1$	
M_n/Ω_b	$\phi_b M_n$
kip-ft	kip-ft
ASD	LRFD

Table 3-10 (continued)

W-Shapes

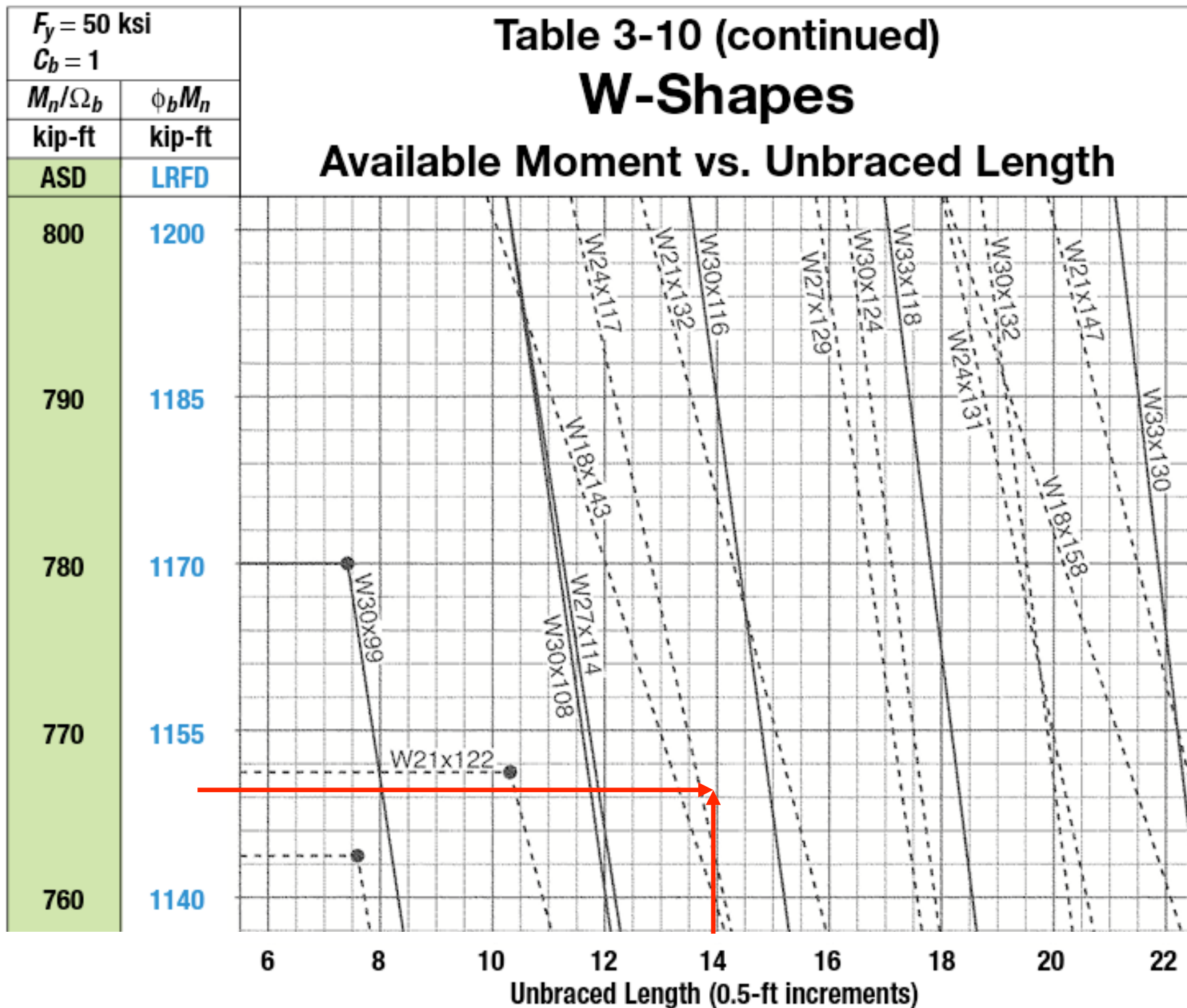
Available Moment vs. Unbraced Length

Available Moment, M_n/Ω_b (2 kip-ft increments) and $\phi_b M_n$ (3 kip-ft increments)



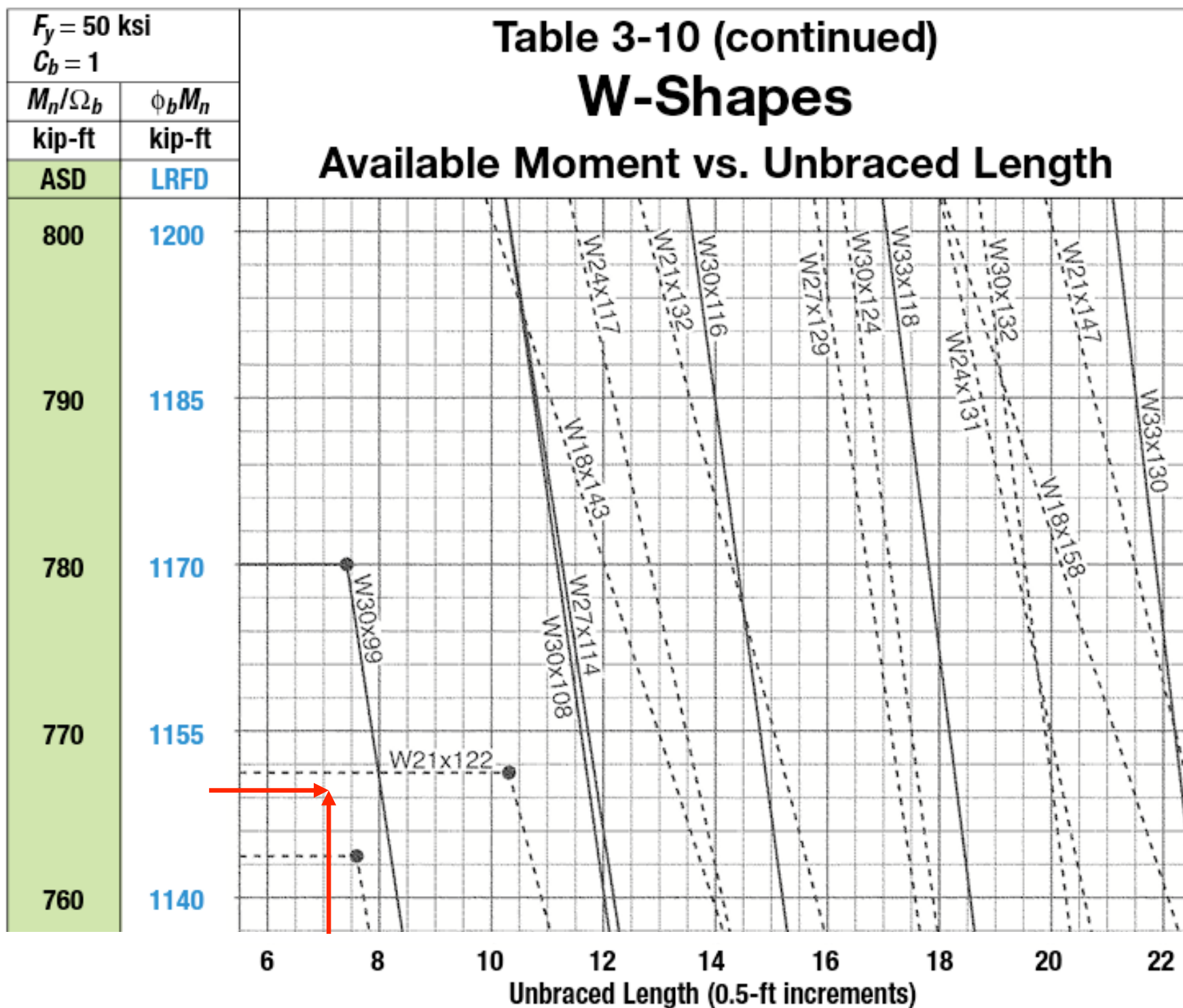
EXAMPLE: $L_b = 14$ ft, $M_u = 1150$ kft, $C_b = 1.0$

Available Moment, M_n/Ω_b (2 kip-ft increments) and $\phi_b M_n$ (3 kip-ft increments)



EXAMPLE: $L_b = 7$ ft, $M_u = 1150$ kft, $C_b = 1.0$

Available Moment, M_n/Ω_b (2 kip-ft increments) and $\phi_b M_n$ (3 kip-ft increments)



Available Moment, M_n/Ω_{fb} (2 kip-ft increments) and $\phi_b M_n$ (3 kip-ft increments)

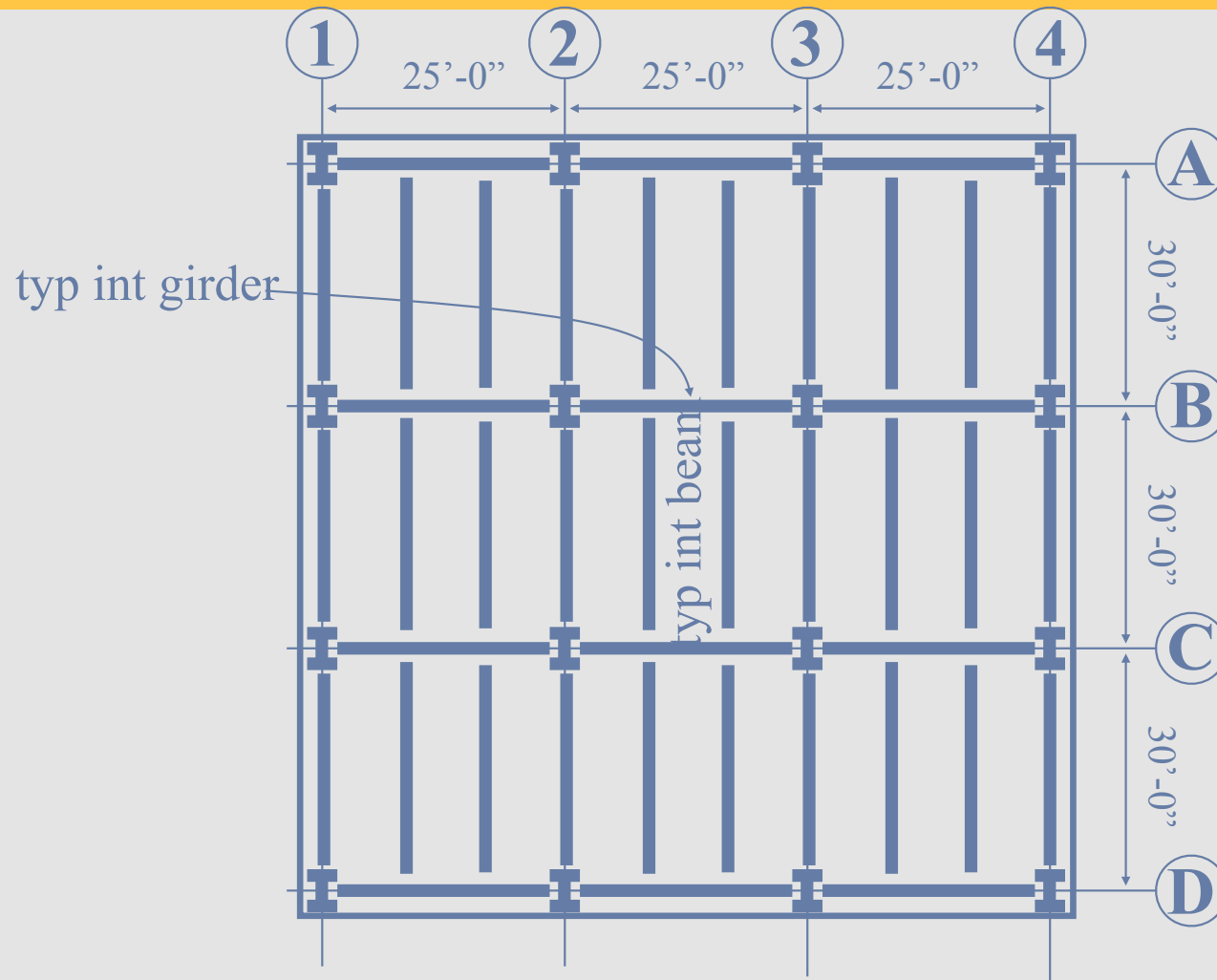
Double check that “real” $M_u < \text{Plastic Moment Capacity}$

Questions?

Example Problem

- Design the most efficient typical interior WF beam.
- Design the most efficient typical interior WF girder.
- Assumptions:
 - Floor Plans are on the following page
 - Floor Loading: Dead = 100psf, Live = 40 psf (reducible)
 - Self weight of the members is included in the dead load
 - A992 Steel
 - Beam compression flanges are unbraced
 - Girder compression flanges are braced only at beam connection locations
 - Deflection Limits: Dead + Live = $L/240$, Live = $L/360$
 - Live Load Reductions: $L = L_0[0.25 + 15/\sqrt{K_{LL} A_T}]$, $L \geq 0.50L_0$ (ASCE 7-05 4.8)

Example Problem



Floor Framing Plan